

Investigation on the Detrimental Wavelength of Track Irregularity for the Suspended Monorail Vehicle System

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Abstract: Track irregularity is a crucial factor influencing the running safety and ride comfort of the suspended monorail-vehicle system. This paper puts an emphasis on the exploration of the detrimental wavelengths of track irregularities for this system. Firstly, the suspended monorail vehicle model with 52 degrees of freedom is modelled by adopting multi-body dynamics theory, where the nonlinear characteristics of tyre and stop structure are considered. Based on the random nature of vehicle-track interactions, the sixth grade track irregularity PSD of U.S. railways is taken as excitation of the driving and guiding wheels in the dynamic computations. Then the characteristic wavelengths are analysed according to the dynamic responses of typical indices of the suspended monorail vehicle by the average smooth periodogram method. On this basis, the coherence theory is employed to emphatically expose the amplitudes-wavelengths relationships between track random irregularities and accelerations of car-body, wheel-rail forces, respectively. Finally, the most sensitive wavelengths of track irregularities can be further determined for the suspended monorail vehicle system. The dynamic results show that the detrimental wavelengths which significantly affect the wheel-rail forces are in the range of 1.5m and 6~17m, and those for the car-body accelerations are mainly about 2m and 6m. This research offers a reference to the maintenance and management of the suspended monorail vehicle-track system.

Keywords: suspended monorail vehicle system; track irregularity; detrimental wavelength; coherence theory; maintenance

1 Introduction

With the gradual improvement of China's city scale and the economic level, the urban population increases rapidly, and the urban traffic becomes more and more crowded. How to effectively solve the congestion status of the people's daily travel is a sophisticated problem needed to be solved urgently in the process of urban sustainable development (Li, 2014). As one of the diversified urban transportation systems, the suspended monorail vehicle has a broad development prospect in China with its advantages of low construction cost, short construction period, low running noise, strong climbing ability and good adaptability for bad weather, etc.

At the same time, track profile irregularity,

which directly affects the wheel-rail interaction force, the running safety and ride comfort (Luo, 1982), is the main excitation source of vehicles and tracks. Therefore, track irregularity is required to be in good status to ensure the safety, stability and comfort of the vehicle in the operation process. It is of great significance to accurately clarify the detrimental wavelengths of the track irregularities for planning reasonable maintenance strategy.

At present, some scholars have conducted lots of researches on this issue. Gao and Yang adopted the harmonic irregularities with different wavelengths at the constant amplitude as the system excitation, and then determined the sensitive wavelengths using the vehicle-track coupled dynamics (Gao et al, 2012; Yang, 2011). Huang and Lian obtained the most detrimental

wavelengths through a coherence function between dynamic responses of vehicle and the track irregularity (Huang et al, 2003; Lian et al, 2006).

By adopting the suspended monorail vehicle model which is established through the multi-body dynamics software of UM, this paper studies the relationship between the responses of dynamic indices of the vehicle and the track irregularities by means of combining the average smooth periodogram method and a coherence theory based on wavelet transform, from which the detrimental wavelengths of track irregularity can be properly determined.

2 Formation of the model

2.1 Introduction of the suspended monorail vehicle structure

The suspended monorail vehicle system mainly consists of the carbody, bogie, hanging systems and the track beam system. The bogie operates in the track beam, of which the shape is a cabinet with placket in the lower part. The carbody is hung under the two bogies through the hanging devices. The wheel/rail interaction between the bogie and track is conducted by the driving and guide wheels, which run respectively on the track surface at the bottom and the side within the track beam in order to achieve the driving and guide functions. Rubber tyre is used in the vehicle system, and the air spring device is employed between the bolster and bogie frame. The hanging system is composed of a center pin, a carbody connecting device and two hoisting beams with a certain inclination at both the right and left sides. A stop device is designed between the center pin and bogie frame, and one anti-yaw damper and two stop devices are also considered between the hoisting beam and carbody connecting device.

2.2 Dynamic model of the suspended monorail vehicle

The suspended monorail vehicle is modelled using 52 degrees of freedoms (DOFs) multi-body system through the multi-body dynamics software of UM, which is composed of a carbody and two bogies. The carbody, bogie frame and bolster are all considered as a rigid body by considering the longitudinal, lateral, vertical displacements, roll, yaw and pitch

motions with 6 DOFs. The center pin presents the rotation motion about the z axis relative to bolster, and the hoisting beam presents the rotation motion about the x axis relative to the center pin. The degrees of freedom of suspended monorail vehicle model are presented in Table 1. The air spring device, stop devices and anti-yaw damper are all treated as spring-damper elements. For the tyre model, not only the stiffness and damping in the radial direction but also the characteristics of cornering behaviours and longitudinal slip are required to take into consideration. The Packjka model of UM software is applied in this simulation model, developed by Hans B. Pacejka, which is a formula named as Magic Formula. The formula can be written as

$$Y(x) = A \sin\{B \arctan[Cx - \arctan(Dx)]\} \quad (1)$$

the dynamic characteristics of tyre can be simulated veritably by selecting appropriately the value of variable,

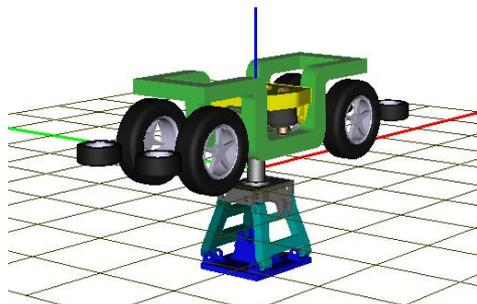


Fig.1 the bogie dynamic model of the suspended monorail vehicle

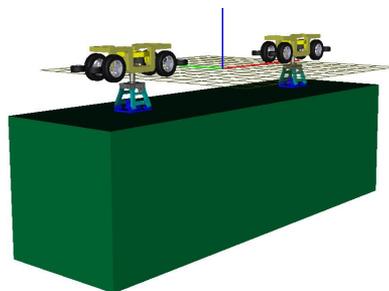


Fig.2 the dynamic model of the suspended monorail vehicle

The sixth grade track irregularity PSD of U.S. railways is taken as excitation of the driving and

guiding wheels in the process of dynamic simulations, the wavelength range of which is from 1m to 30m. The vehicle running speed is 50km/h. The dynamic responses of typical indices of the suspended monorail vehicle is

researched and analysed in this paper, such as the wheel-rail vertical force of the driving wheel, the wheel-rail lateral force of the guiding wheel, the vertical and lateral vibration acceleration of carbody.

Table 1 The degrees of freedom in the dynamic model of suspended monorail vehicle system

| | LONG. Displacement | Lateral Displacement | Vertical Displacement | Roll Motion | Pitch Motion | Yaw Motion |
|-------------------------------|-----------------------|-------------------------|--------------------------|----------------|-----------------|---------------|
| Carbody ($i=1$) | x_c | y_c | z_c | θ_c | ψ_c | ϕ_c |
| Frame ($i=1,2$) | x_f | y_f | z_f | θ_f | ψ_f | ϕ_f |
| Bolster ($i=1,2$) | x_{bi} | y_{bi} | z_{bi} | θ_{bi} | ψ_{bi} | ϕ_{bi} |
| Center Pin ($i=1,2$) | — | — | — | — | — | ϕ_{cpi} |
| Hoisting beam($i=1\sim 4$) | — | — | — | θ_{hbi} | — | — |
| Driving Wheel($i=1\sim 8$) | — | — | — | — | ψ_{dwi} | — |
| Guiding Wheel ($i=1\sim 8$) | — | — | — | — | — | ϕ_{gwi} |

3 Average smooth periodogram method

The power spectral density function is one of the basic method for frequency analysis of the simulated results. In this paper, the average smooth periodogram method, which is different from the linear discrete Fourier transform (DFT) method, is employed. The principle of basic periodogram method is to directly carry out the Fourier transform directly on the observed data, then the square of norm is power spectrum. For a finite number of observation points $x(0), x(1) \dots x(n-1)$ from a random signal $x(n)$, the Fourier transform can be defined by

$$X_N(e^{-j\omega}) = \sum_{n=0}^{N-1} x(n)e^{-j\omega n} \tag{2}$$

then the power spectrum can be estimated by

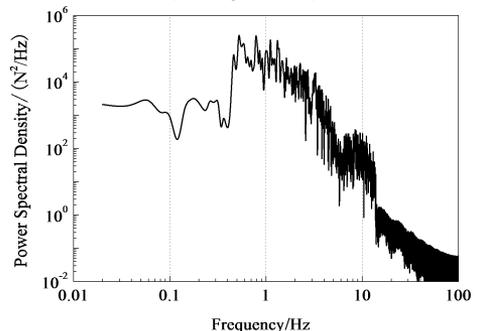
$$\hat{P}(\omega) = \frac{1}{N} \left| X_N e^{-j\omega n} \right|^2 = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n)e^{-j\omega n} \right|^2 \tag{3}$$

In order to reduce the random fluctuation, M.S. Bartlett proposed the average periodogram method, in which the signal sequence $x(n)$ with N discrete points is divided into L segments. Then each segment with a length of M is

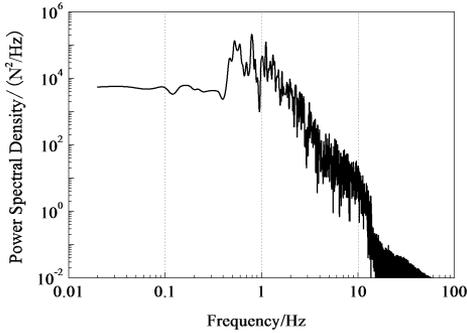
calculated by the basic periodogram method. The average results of the calculation for each segment is namely the estimation of the power spectrum for the signal $x(n)$, which can be expressed as

$$\hat{P} = \frac{1}{ML} \sum_{l=1}^L \left| \sum_{n=0}^M x_m^l(n) e^{-j\omega n} \right|^2 \tag{4}$$

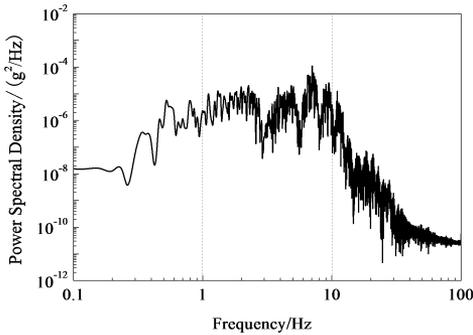
Compared with the traditional Fourier transform, this method can reduce the random fluctuation and provide the prominent features of the results with better convergence, curve smoothing, and smaller variance (Huang, 2013).



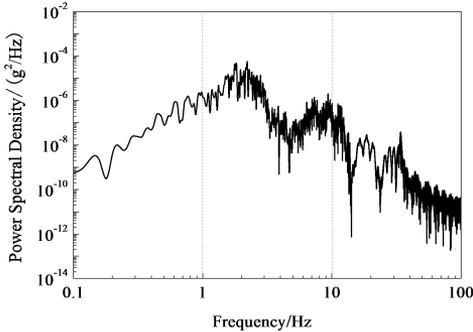
(a) PSD of the wheel-rail vertical force of the driving wheel



(b) PSD of the wheel-rail lateral force of the guiding wheel



(c) PSD of the vertical acceleration of carbody



(d) PSD of the lateral acceleration of carbody
Fig. 3 PSD of dynamic responses of the typical indices

Figure 3 shows the PSD of dynamic responses of the typical indices for the suspended monorail vehicle. There are two dominating vibration frequencies to the wheel-rail vertical force of the driving wheel, as shown in Fig.3(a). The first frequency part mainly distributed in low frequency of about 0.8~1.6Hz, and another part mostly focuses on

midfrequency of 8~10Hz, the characteristic wavelengths approximately is, therefore, 1.4~1.8m and 8.6~17.3m. As for the wheel-rail lateral force of the guiding wheel, the driving frequency is about 0.8~1.6Hz, to which the wavelength corresponding about 8.6~17.3m. By comparing Fig.3 (c) with (d), the principal vibration frequency of the vertical and lateral acceleration of carbody is 6.5~9.6Hz and 1.7~2.4Hz. The corresponding wavelength is about 1.4~2.1m and 5.7~8.2m. The principal vibration frequency of the vertical acceleration is higher than that of the lateral acceleration. The results are basically the same as those in references (Zhai, 2015; Garg et al, 1984). Besides, the results in Fig.3 (d) also have another peak at the frequency of 7Hz. The relevant wavelength is about 2m.

4 Coherence theory

The coherence theory can be used to determine the extent to which the dynamic response of the vehicle is derived from the input disturbance (An et al, 2010; Li et al, 2014). Based on this, the coherence between the dynamic response of the suspended monorail vehicle and track irregularities can be analysed to determine the characteristic wavelengths which have the greatest influence on the vehicle vibration.

For a complete system, the correlation between input and output is often analysed by coherent functions(CF). The coherent function between the stationary random signal $x(t)$ and $y(t)$ is defined as

$$r_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f) \cdot G_{yy}(f)} \quad (5)$$

where $G_{xx}(f)$ and $G_{yy}(f)$ are respectively the unilateral self-power spectrum of $x(t)$ and $y(t)$, and $G_{xy}(f)$ is the unidirectional cross-power spectral density function of $G_{xx}(f)$ and $G_{yy}(f)$. The larger the value of $r_{xy}^2(f)$, which varies from zero to one, the stronger coherence between $x(t)$ and $y(t)$.

The wavelet coherence is derived from the FFT coherent, which can be written as

$$(WCo(t, f))^2 = \frac{|SW_{xy}(t, f)|^2}{SW_{xx}(t, f) \cdot SW_{yy}(t, f)} \quad (6)$$

with

$$SW_{xy}(t, f) = \int_{t-\delta/2}^{t+\delta/2} W_x(\tau, f) \cdot W_y(\tau, f) d\tau \quad (7)$$

Besides, $SW_{xx}(t, f)$ and $SW_{yy}(t, f)$ can also be calculated by formula (7); δ varies with the frequency required to be focused on, whose value can be obtained according to the following formula:

$$\delta = n_{cy}/f \quad (8)$$

where n_{cy} is the number of periods in the time period. This expression reflects the basic idea of wavelet coherence, namely, it uses a narrower integral window for higher frequencies. Moreover, morlet wavelet is introduced in this paper for the advantage of good time clustering and high frequency resolution (Wu, 2007).

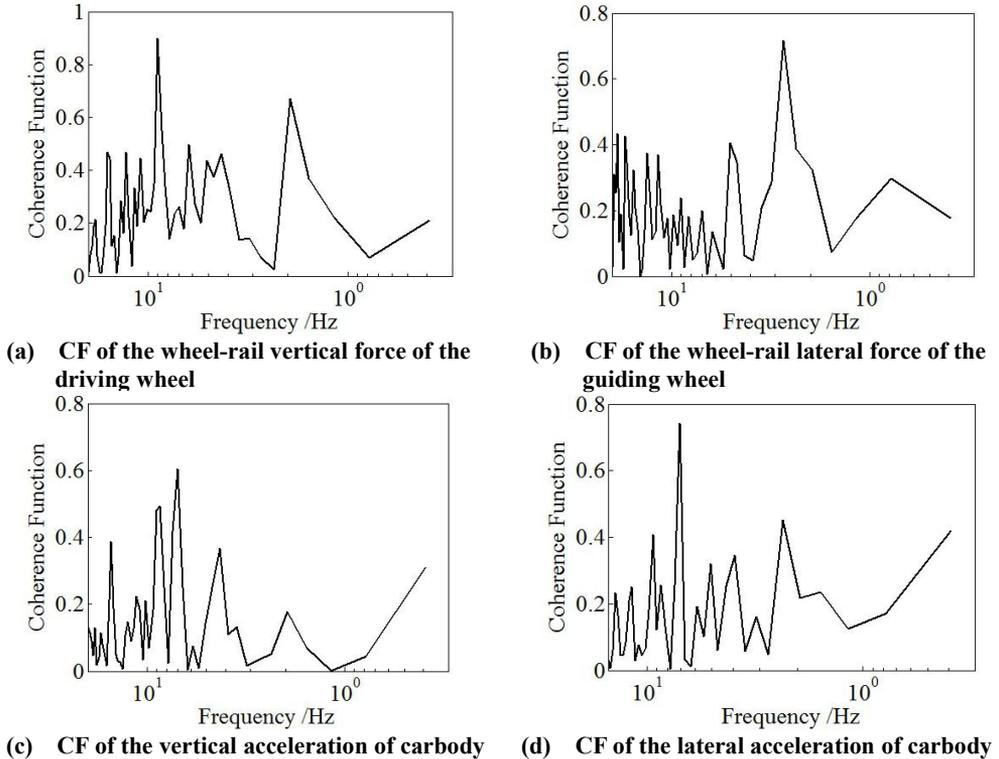


Fig. 4 Coherence functions of dynamic responses for the typical indices

Figure 4 gives the coherence functions of the dynamic responses of the typical indices and the track surface irregularity. If there is a maximum value of the coherence functions at a certain frequency, it can be considered that this frequency is the most detrimental to the vibration of the suspended monorail vehicle system. From the Fig. 4 (a), the feature frequencies of the wheel-rail vertical force are about 1.8Hz and 8.9Hz, so the detrimental wavelengths are 7.72m and 1.56m. As for the results of the wheel-rail lateral force of guiding wheel presented as Fig. 4(b), there is only one peak value at the sensitive frequency of 2~3Hz, of which the detrimental wavelength is about 5.79m. Fig. 4(c) and (d) offers the coherence function of the vertical and

lateral acceleration of the carbody. It can be drawn that the sensitive frequency of the vertical acceleration is 7.5Hz and the desired detrimental wavelength is about 1.85m. Meanwhile, the characteristic frequency of the lateral acceleration is about 2.2Hz and 7.2Hz, and the relevant wavelengths are 1.92m and 6.31m.

5 Conclusions

A dynamic model of the suspended monorail vehicle is established based on the multi-body dynamic theory. The average periodogram method and wavelet coherence analysis are employed to investigate the detrimental

wavelengths of track irregularity. Based on the results of the above two methods, which have quite good consistency, the following conclusions can be drawn:

- (1) Compared with the traditional method, wavelet coherent can compensate some shortcomings and carry out an effective analysis on the relationship between the track irregularities and the vehicle dynamic responses.
- (2) the detrimental wavelengths of the track irregularity which significantly affect the wheel-rail vertical forces are about 1.56m and 7.72m, and which influence the wheel-rail lateral forces are 5.79m and 8.6~17.3m.
- (3) the detrimental wavelength of the track irregularity which significantly affect the vertical acceleration of carbody is about 1.85m, and which have a great impact on the lateral acceleration of carbody is about 1.92m and 6.31m.

6 Acknowledgement

This work was supported by the Sichuan Science and Technology Plan Project (Grant No. 2017GZ20082).

References

- An M. J., Li S. M., and Zhang Y. Y., (2010). "Coherence Analysis of Inputs Using Multiple Correlated Subsystem-inputs System Modeling." *China Mechanic Engineering*, 21(21), 2609-2613.
- Gao J. M., Zhai W. M., and Wang K. Y., (2012). "Study on Sensitive Wavelengths of Track Irregularities in High-speed Operation". *Journal of the China Railway Society*, 34(7), 83-88.
- Garg V. K., and Dukkipati R. V., (1984). "Dynamics of Railway Vehicle Systems". Academic Press. Ontario.
- Huang J. F., Lian S. L., and Zong D. M., (2003). "Analysis of Coherence between Track Random Irregularity and Vehicle Dynamic Response." *Journal of Tongji University*, 31(1), 16-19.
- Huang Y., (2013). "Power spectrum estimation based on Periodogram". *Science and Technology of West China*, 12(9), 1-3.
- Li, F., and Xu W., (2014). "Development and Current Status of Suspended Monorail Vehicle." *Electric Drive for Locomotives*, 2, 16-20.
- Li X. Z., Liu X. H., Zhang X., and Li Y. D., (2014). "Research on Identification of Structure-Borne Noise Source of High-Speed Railway Simply-Supported Box Girder Based on Coherence Analysis." *Engineering Mechanics*, 31(1), 129-136.
- Lian S. L., Lu H. M., and Yang W. Z., (2006). "Program for Rail Irregularity Analysis." *China Railway Science*, 27(1), 68-71.
- Luo L., (1982). "Track random interference function". *China Railway Science*, 1, 79-117.
- Wu J., Zhang N., Yang Z., and Zhang T., (2007). "Wavelet Coherence and Its Application in Analyzing Auditory and Motor Task Event-related Potentials." *Acta Biophysica Sinica*. 23(6), 482-487.
- Yang F., (2011). "A study of sensitive wavelength and the management value of Long-wave irregularity for high-speed railway." Master Degree. China Academy of Railway Sciences.
- Zhai. W. M., (2015). "Vehicle-Track Coupled Dynamics (Fourth Edition)" Volume 1. Science Press. Beijing.